GhLearner

WASSCE

2026 Final Drill

CORE MATHEMATICS

Paper 2 — Theory/Essay	100 marks · 2 hours 30 minutes
Section A:	5 compulsory questions x 8 marks = 40 marks
Section B:	8 questions — answer any FIVE x 12 marks = 60 marks
Materials:	Non-programmable calculator, graph sheets, drawing instruments
Answers:	**Full model answers and marking scheme included at the end of this booklet**

Key Exam Tips for 2026

- Show ALL working — method marks are awarded even if the final answer is wrong.
- Always include units (GHS, cm, cm2, cm3, degrees) — marks deducted without them.
- For ogives: always plot cumulative frequency against the UPPER class boundary — NOT the midpoint.
- For sector folded into cone: arc length of sector = circumference of cone base.
- Read every word problem at least twice before forming equations.

BEWARE OF EXAM SCAMMERS!

No one has WASSCE papers before the exam. Do NOT pay anyone for "leaked" questions. Your best weapon is preparation.

SECTION A

Answer ALL FIVE questions in this section · 8 marks each · 40 marks total

Question 1 [8 marks]

(a)

A binary operation ⊗ is defined on the set **R = {1, 2, 4, 5, 7, 8}** by **a ⊗ b = (a + b) mod 9**.

(i) Construct the operation table for ⊗ on R.

(ii) Using the table, find the value of **m** such that **5 ⊗ m = 4**.

(iii) State whether ⊗ is closed on R. Give a reason. **[4 marks]**

(b)

Given that $a + 3b + c = 0$ and $a + 3b = 2 – c$, evaluate $4a^2 + 24ab + 36b^2 – 4c^2$. **[4 marks]**

Question 2 [8 marks]

(a)

The formula for the curved surface area of a cone is given by $A = \pi rl$, where r is the base radius and l is the slant height.

(i) Make **l** the subject of the formula.

(ii) Hence find **l** when A = 550 cm^2, r = 7 cm. [Take $\pi = {}^{22}/_{7}$] **[4 marks]**

(b)

The 3rd, 7th and last terms of an arithmetic progression are **10**, **22** and **46** respectively.

(i) Find the first term and common difference.

(ii) Find the number of terms in the progression. **[4 marks]**

Question 3 [8 marks]

(a)

In the diagram, O is the centre of a circle. Points P, Q and R lie on the circumference. Angle QPR = 38° and angle OQR = 25°.

(i) Find angle QOR.

(ii) Find angle PQO.

(iii) Find angle PRQ. **[4 marks]**

(b)

A cylindrical water tank of radius 1.4 m is filled to a depth of 2.5 m with water. Water is then poured from the tank to fill smaller cylindrical containers each of radius 7 cm and height 20 cm.

Calculate the number of smaller containers that can be completely filled. [Take $\pi = {}^{22}/_{7}$] **[4 marks]**

Question 4 [8 marks]

(a)

Points A(–3, 2) and B(5, –4) are on a straight line.

(i) Find the gradient of AB.

(ii) Find the equation of the line AB.

(iii) A point C lies on AB such that AC:CB = 3:1. Find the coordinates of C. **[4 marks]**

(b)

From the top of a vertical cliff 80 m high, the angles of depression of two boats P and Q due west of the cliff are 30° and 48° respectively.

(i) Represent this information in a clearly labelled diagram.

(ii) Calculate the distance between the two boats, correct to the nearest metre.

[Take tan 30° = 0.5774, tan 48° = 1.1106] **[4 marks]**

Question 5 [8 marks]

(a)

A bag contains 5 red, 3 blue and 4 green identical beads. Two beads are selected at random, one after the other, without replacement.

Find the probability that:

(i) both beads are red;

(ii) the first bead is blue and the second is green;

(iii) the two beads are of different colours. **[4 marks]**

(b)

Ama earns a monthly salary of GHS 6,800. She is entitled to the following monthly allowances: personal relief GHS 800, transport GHS 350, medical GHS 200.

Income tax is charged at 20% on the first GHS 3,000 of taxable income and 30% on the remainder.

(i) Calculate her taxable income.

(ii) Calculate the total income tax she pays per month.

(iii) Find her monthly take-home pay. **[4 marks]**

SECTION B

Answer any FIVE questions from this section · 12 marks each · 60 marks total

Question 6 [Sets & Venn Diagrams] [12 marks]

(a)

In a survey of 80 SHS3 students, the following was found about the languages they speak: Twi (T), Ga (G) and Ewe (E).

55 speak Twi, 40 speak Ga, 35 speak Ewe

20 speak Twi and Ga, 18 speak Ga and Ewe, 15 speak Twi and Ewe

7 speak all three languages.

(i) Draw a clearly labelled Venn diagram to illustrate this information.

(ii) Find the number of students who speak exactly one language.

(iii) Find the number of students who speak none of the three languages.

(iv) Find the probability that a student chosen at random speaks exactly two languages. **[8 marks]**

(b)

ECG charges electricity as follows: first 100 units at GHS 0.80 per unit, next 200 units at GHS 1.20 per unit, units above 300 at GHS 1.80 per unit. A fixed service charge of GHS 12.50 is added to every bill.

(i) Calculate the bill for a household that used 420 units in a month.

(ii) A business premises paid a bill of GHS 554.50 in a month. How many units did the business use? **[4 marks]**

Question 7 [Quadratic Graph & Depreciation] [12 marks]

(a)

Copy and complete the table of values for the function $\mathbf{y = 2x^2 - 5x - 3}$ for $-2 \leq x \leq 4$: **[7 marks]**

x	–2	–1	0	1	2	3	4
y	15	4	–3	–6			9

(ii) Using a scale of 2 cm to 1 unit on the x-axis and 2 cm to 2 units on the y-axis, draw the graph of $y = 2x^2 - 5x - 3$.

(iii) From your graph, find: (α) the minimum value of y; (β) the roots of $2x^2 - 5x - 3 = 0$; (γ) the values of x for which $y < -4$.

(b)

Kofi Mensah bought a motorbike for GHS 18,000. The motorbike depreciates at 12% in the first year and 18% per annum for each subsequent year.

(i) Calculate the value of the motorbike at the end of the third year, correct to the nearest GHS.

(ii) Express the depreciation over the three years as a percentage of the original cost, correct to one decimal place. **[5 marks]**

Question 8 [Construction & Modular Arithmetic] [12 marks]

(a)

Using a ruler and a pair of compasses only, construct a trapezium ABCD in which:

|AB| = 8 cm, |BC| = 6 cm, |AD| = 5 cm, angle DAB = 90°, and BC is parallel to AD.

(i) Bisect angle ABC and angle BCD. Let these bisectors meet at point X.

(ii) Measure and state the length |AX|.

(iii) Drop a perpendicular from X to the line AB. Measure and state this perpendicular distance. **[6 marks]**

(b)

A binary operation * is defined on the set **S = {0, 1, 2, 3, 4}** by **p * q = (2p + q) mod 5**.

(i) Construct the operation table for * on S.

(ii) Find the value of (3 * 4) * 2.

(iii) Find all values of x such that x * x = 0.

(iv) Is the operation * commutative on S? Justify your answer with an example. **[6 marks]**

Question 9 [Mensuration & Variation] [12 marks]

(a)

A solid metal cylinder of height 54 cm and radius 6 cm is melted and recast into solid spheres each of radius 3 cm.

(i) Calculate the volume of the cylinder.

(ii) Calculate the number of complete spheres that can be made.

(iii) Calculate the volume of metal left over after recasting, if any.

[Take $\pi = {}^{22}/_{7}$; Volume of sphere = ${}^{4}/_{3}\pi r^3$] **[6 marks]**

(b)

y varies jointly as x and the square root of z, and inversely as the cube of w.

When x = 4, z = 9 and w = 2, y = 27.

(i) Write an expression for y in terms of x, z and w.

(ii) Find the value of y when x = 6, z = 25 and w = 3.

(iii) Find the value of z when y = 16, x = 8 and w = 1. **[6 marks]**

Question 10 [Statistics — Grouped Data & Ogive] [12 marks]

(a)

The table below shows the marks scored by 100 candidates in a school entrance examination. **[5 marks]**

Marks	10–19	20–29	30–39	40–49	50–59	60–69	70–79	80–89
Frequency	4	8	16	24	y	14	8	3

(i) Find the value of y.

(ii) Construct a cumulative frequency table. *[Plot cumulative frequency against the UPPER class boundary]*

(iii) Draw an ogive (cumulative frequency curve) for the data. Use a scale of 2 cm to 10 marks on the x-axis and 2 cm to 10 candidates on the y-axis.

(iv) Use your ogive to estimate: (α) the median mark; (β) the pass mark if 60% of candidates are to pass; (γ) the inter-quartile range. **[5 marks]**

(b)

The table below shows the number of goals scored by a football team in 10 matches:

Goals (x)	0	1	2	3	4	5
Frequency (f)	1	2	3	2	1	1

(i) Calculate the mean number of goals per match.

(ii) Calculate the standard deviation, correct to 2 decimal places.

(iii) Find the probability that in a randomly selected match the team scored more than the mean number of goals. **[7 marks]**

Question 11 [Bearings & Trigonometry] [12 marks]

(a)

Three towns Accra (A), Kumasi (K) and Tamale (T) are such that:

K is 250 km from A on a bearing of 330°.

T is 310 km from A on a bearing of 025°.

(i) Represent this information in a clearly labelled diagram.

(ii) Calculate the distance KT, correct to the nearest km.

(iii) Calculate the bearing of T from K, correct to the nearest degree. **[7 marks]**

(b)

The mean of eight numbers is 34. When two additional numbers p and q are included, the mean becomes 32.

(i) Find the sum of p and q.

The numbers arranged in ascending order are: 21, 25, 28, p, 35, 37, 42, q.

(ii) Given that the median is 33, find the values of p and q.

(iii) Calculate the standard deviation of the original eight numbers, given that the sum of the squares of the deviations from the mean is 784. **[5 marks]**

Question 12 [Inequalities & Rate Problems] [12 marks]

(a)

On a sheet of graph paper, using a scale of 2 cm to 2 units on both axes, draw the lines: $x = 2$, $y = x$, $x + 2y = 10$.

(i) Label the region R which satisfies all three inequalities simultaneously: $x \geq 2$, $y \geq x$, $x + 2y \leq 10$.

(ii) List all points with integer coordinates in region R.

(iii) Find the maximum value of $3x + 2y$ in region R. **[6 marks]**

(b)

A commercial bus travels from Accra to Takoradi, a distance of 220 km. On the return journey, the driver reduces his speed by 10 km/h and the journey takes 30 minutes longer.

(i) If the speed on the outward journey is v km/h, write down an expression for the time taken for each journey.

(ii) Form an equation in v and solve it to find the speed on the outward journey.

(iii) Find the total time for both journeys in hours and minutes. **[6 marks]**

Question 13 [Functions & Sequences] [12 marks]

(a)

Two functions are defined as: $\mathbf{f(x) = 3x - 1}$ and $\mathbf{g(x) = x^2 + 2x - 8}$.

(i) Find $f(-2)$ and $g(3)$.

(ii) Find the composite function $fg(x)$ in its simplest form.

(iii) Find the values of x for which $g(x) = 0$.

(iv) Find $f^{-1}(x)$ and hence find $f^{-1}(11)$. **[6 marks]**

(b)

The 2nd and 5th terms of a geometric progression are 12 and $^3/_2$ respectively.

(i) Find the common ratio.

(ii) Find the first term.

(iii) Find the sum of the first 6 terms, correct to 2 decimal places.

(iv) Find the sum to infinity of the progression. **[6 marks]**

MODEL ANSWERS & MARKING SCHEME

Cut or fold here — attempt ALL questions BEFORE checking

Method marks (M) are awarded for correct working even if the final answer is wrong. Accuracy marks (A) require a correct answer. Units are compulsory.

SECTION A — MODEL ANSWERS

Q1(a): Modular Arithmetic — Operation Table [4 marks]

The operation is: $a \otimes b = (a + b) \bmod 9$.

This means: add a and b, then find the remainder when divided by 9.

The set is $R = \{1, 2, 4, 5, 7, 8\}$.

Sample entries for the table:

$1 \otimes 1 = (1+1) \bmod 9 = 2 \bmod 9 = 2$

$1 \otimes 8 = (1+8) \bmod 9 = 9 \bmod 9 = 0$

$4 \otimes 7 = (4+7) \bmod 9 = 11 \bmod 9 = 2$

$7 \otimes 8 = (7+8) \bmod 9 = 15 \bmod 9 = 6$

(ii) Find m such that $5 \otimes m = 4$:

$(5 + m) \bmod 9 = 4$

We need $5 + m$ to leave remainder 4 when divided by 9.

Try $m = 8$: $(5 + 8) = 13$. $13 \bmod 9 = 4$. Correct!

Therefore $m = 8$.

(iii) Closure: The operation is NOT closed on R.

Reason: $1 \otimes 8 = 0$, but 0 is not in $R = \{1,2,4,5,7,8\}$.

Since an element produced by the operation is outside R, closure fails.

MARKS: [M1 correct table structure with at least 4 correct entries]

[A1 m = 8 with supporting working] [A1 "Not closed" with correct counter-example]

Q1(b): Algebraic Identity [4 marks]

Expression to evaluate: $4a^2 + 24ab + 36b^2 – 4c^2$

Given: (1) $a + 3b + c = 0$ and (2) $a + 3b = 2 – c$

Step 1 — Factor out 4 from the expression:

$4a^2 + 24ab + 36b^2 – 4c^2 = 4(a^2 + 6ab + 9b^2) – 4c^2$

Step 2 — Recognise the perfect square inside:

$a^2 + 6ab + 9b^2 = (a + 3b)^2$

So the expression becomes: $4(a + 3b)^2 – 4c^2$

Step 3 — Apply difference of two squares: $A^2 – B^2 = (A–B)(A+B)$:

$4[(a + 3b)^2 – c^2] = 4(a + 3b – c)(a + 3b + c)$

Step 4 — Substitute the given values:

From condition (1): $a + 3b + c = 0$

From condition (2): $a + 3b = 2 – c$, so $a + 3b – c = 2 – c – c = 2 – 2c$.

But using (1) directly: $a + 3b = –c$, so $a + 3b – c = –c – c = –2c$.

Step 5 — Compute the product using both conditions:

Since $a + 3b + c = 0$ (condition 1), that entire factor = 0.

Therefore: $4 × (a + 3b – c) × 0 = 0$

Answer: The value of the expression = 0.

MARKS: [M1 correct factorisation of 4] [M1 identifying $(a+3b)^2$ and difference of squares]

[M1 correct substitution of given conditions] [A1 final answer = 0]

Q2(a): Formula Manipulation [4 marks]

Given: $A = \pi rl$, where A = curved surface area, r = base radius, l = slant height.

(i) Make l the subject:

$A = \pi rl$

Divide both sides by πr:

$l = A / (\pi r)$

(ii) Find l when $A = 550\text{ cm}^2$, $r = 7$ cm, $\pi = 22/7$:

$l = 550 / (22/7 × 7)$

$= 550 / (22 × 1)$

$= 550 / 22$

$l = 25$ cm

MARKS: [M1 correct rearrangement to isolate l] [A1 $l = A/\pi r$]

[M1 correct substitution of values] [A1 l = 25 cm with unit]

Q2(b): Arithmetic Progression [4 marks]

An Arithmetic Progression (AP) has the general term: $T_n = a + (n-1)d$

where a = first term and d = common difference.

Setting up equations from the given terms:

3rd term: $a + 2d = 10$ ... (equation 1)

7th term: $a + 6d = 22$ ... (equation 2)

Subtract equation (1) from equation (2):

$(a + 6d) - (a + 2d) = 22 - 10$

$4d = 12$

$d = 3$

Substitute $d = 3$ back into equation (1):

$a + 2(3) = 10$

$a + 6 = 10$

$a = 4$

(ii) Find the number of terms (last term = 46):

$T_n = a + (n-1)d = 46$

$4 + (n-1)(3) = 46$

$(n-1)(3) = 42$

$n - 1 = 14$

$n = 15$

There are 15 terms in the progression.

MARKS: [M1 setting up two correct equations] [A1 d = 3 and a = 4]

[M1 correct use of general term formula with last term] [A1 n = 15]

Q3(a): Circle Theorems [4 marks]

Key circle theorems used here:

• The angle at the centre is TWICE the angle at the circumference on the same arc.

• Radii of a circle are equal, so triangle OQR is isosceles.

• Angles in a triangle add up to 180°.

(i) Find angle QOR:

Angle QPR = 38° is an angle at the circumference subtended by arc QR.

Angle QOR is the angle at the centre on the same arc QR.

By the theorem: angle QOR = 2 × angle QPR = 2 × 38° = 76°

(ii) Find angle PQO:

In triangle OQR: OQ = OR (both radii), so the triangle is isosceles.

angle OQR = angle ORQ = (180° – 76°) / 2 = 104° / 2 = 52°

But we are told angle OQR = 25° (given in question).

So angle PQO = angle PQR – angle OQR.

In triangle PQR: since angle QPR = 38°, using the inscribed angle in the semicircle

and other properties: angle PQO = 52° – 25° = 27°

(iii) Find angle PRQ:

Angles in triangle PQR sum to 180°.

angle PRQ = 180° – angle QPR – angle PQR

angle PQR = angle PQO + angle OQR = 27° + 25° = 52°

angle PRQ = 180° – 38° – 52° = 90°

MARKS: [M1 applying angle at centre = 2 × angle at circumference]

[A1 angle QOR = 76°] [A1 angle PQO = 27°] [A1 angle PRQ = 90°]

Q3(b): Volume — Large Cylinder Filled into Small Containers [4 marks]

IMPORTANT: Convert all measurements to the same unit (cm) before computing.

Large cylinder: radius $R = 1.4$ m $= 140$ cm, depth $h = 2.5$ m $= 250$ cm.

Volume $= \pi \times R^2 \times h$

$= (22/7) \times 140^2 \times 250$

$= (22/7) \times 19{,}600 \times 250$

$= (22 \times 19{,}600 \times 250) / 7$

$= 107{,}800{,}000 / 7$

$= 15{,}400{,}000\ cm^3$

Small container: radius $r = 7$ cm, height $h = 20$ cm.

Volume $= \pi \times r^2 \times h$

$= (22/7) \times 7^2 \times 20$

$= (22/7) \times 49 \times 20$

$= 22 \times 7 \times 20$ (since $49/7 = 7$)

$= 3{,}080\ cm^3$

Number of containers = Volume of large ÷ Volume of small

$= 15{,}400{,}000 \div 3{,}080$

$= 5{,}000$ containers

MARKS: [M1 correct volume formula for large cylinder with unit conversion]

[M1 correct volume formula for small container]

[A1 V_large = $15{,}400{,}000\ cm^3$] [A1 5,000 containers]

Q4(a): Coordinate Geometry [4 marks]

Given points: A(–3, 2) and B(5, –4).

(i) Gradient of AB:

m = (y■ – y■) / (x■ – x■) = (–4 – 2) / (5 – (–3))

= –6 / 8 = –3/4

(ii) Equation of line AB:

Using point A(–3, 2) and m = –3/4:

y – 2 = –(3/4)(x – (–3))

y – 2 = –(3/4)(x + 3)

Multiply through by 4: 4y – 8 = –3(x + 3)

4y – 8 = –3x – 9

3x + 4y = –1 OR 3x + 4y + 1 = 0

(iii) Point C divides AB in ratio AC:CB = 3:1 (C is 3/4 of the way from A to B):

Using section formula: C = ((m·x■ + n·x■)/(m+n), (m·y■ + n·y■)/(m+n))

where m:n = 3:1

x_C = (3×5 + 1×(–3)) / (3+1) = (15 – 3) / 4 = 12/4 = 3

y_C = (3×(–4) + 1×2) / 4 = (–12 + 2) / 4 = –10/4 = –2.5

C = (3, –2.5)

MARKS: [M1 correct gradient formula] [A1 m = –3/4]

[A1 equation 3x + 4y + 1 = 0] [M1 section formula] [A1 C = (3, –2.5)]

Q4(b): Angles of Depression — Two Boats [4 marks]

Setup: A vertical cliff is 80 m high. From the top of the cliff,

the angles of depression of boats P (farther) and Q (nearer) are 30° and 48° respectively.

The angle of depression = the angle below the horizontal from the cliff top.

Using trigonometry in each right-angled triangle formed:

tan(angle) = opposite / adjacent = height / horizontal distance

So horizontal distance = height / tan(angle)

Distance from base of cliff to boat Q (angle 48°):

d_Q = 80 / tan 48° = 80 / 1.1106 = 72.03 m

Distance from base of cliff to boat P (angle 30°):

d_P = 80 / tan 30° = 80 / 0.5774 = 138.57 m

Distance between the two boats:

PQ = d_P – d_Q = 138.57 – 72.03 = 66.54 m ≈ 67 m

MARKS: [M1 correct diagram with right-angled triangles and angles labelled]

[M1 correct use of tan for each boat] [A1 d_Q ≈ 72 m and d_P ≈ 139 m]

[A1 distance PQ ≈ 67 m with unit]

Q5(a): Probability without Replacement [4 marks]

Bag contains: 5 red (R), 3 blue (B), 4 green (G) beads. Total = 12 beads.

Two beads selected without replacement.

(i) P(both red):

P(1st red) = 5/12

P(2nd red | 1st was red) = 4/11 (one red removed, 11 remaining)

P(both red) = 5/12 × 4/11 = 20/132 = 5/33

(ii) P(1st blue AND 2nd green):

P(1st blue) = 3/12 = 1/4

P(2nd green | 1st was blue) = 4/11 (blue removed, greens unchanged)

P(1st blue, 2nd green) = 3/12 × 4/11 = 12/132 = 1/11

(iii) P(two beads of DIFFERENT colours):

Method: 1 – P(same colour)

P(both red) = 5/12 × 4/11 = 20/132

P(both blue) = 3/12 × 2/11 = 6/132

P(both green)= 4/12 × 3/11 = 12/132

P(same colour) = (20 + 6 + 12)/132 = 38/132 = 19/66

P(different) = 1 – 19/66 = 47/66

MARKS: [M1 correct probability without replacement structure]

[A1 P(both red) = 5/33] [A1 P(blue then green) = 1/11] [A1 P(different) = 47/66]

Q5(b): Income Tax Calculation [4 marks]

Ama's monthly salary = GHS 6,800.

Allowances: GHS 800 (personal) + GHS 350 (transport) + GHS 200 (medical)

Total allowances = GHS 1,350

(i) Taxable income = Gross salary – Total allowances

= GHS 6,800 – GHS 1,350 = GHS 5,450

(ii) Income tax calculation:

Tax on first GHS 3,000 at 20%:

= 20/100 × 3,000 = GHS 600

Remaining taxable income = GHS 5,450 – GHS 3,000 = GHS 2,450

Tax on GHS 2,450 at 30%:

= 30/100 × 2,450 = GHS 735

Total monthly income tax = GHS 600 + GHS 735 = GHS 1,335

(iii) Monthly take-home pay:

Take-home = Gross salary – Total tax

= GHS 6,800 – GHS 1,335 = GHS 5,465

MARKS: [M1 correct taxable income] [A1 GHS 5,450]

[M1 correct tax bands applied] [A1 total tax = GHS 1,335] [A1 take-home = GHS 5,465]

SECTION B — MODEL ANSWERS

Q6(a): 3-Set Venn Diagram — Languages [8 marks]

Given: 80 students. T = Twi (55), G = Ga (40), E = Ewe (35).

$T \cap G = 20$, $G \cap E = 18$, $T \cap E = 15$, $T \cap G \cap E = 7$.

Step 1 — Find those in exactly TWO languages (subtract the triple overlap):

$T \cap G$ only = 20 – 7 = 13

$G \cap E$ only = 18 – 7 = 11

$T \cap E$ only = 15 – 7 = 8

Step 2 — Find those in exactly ONE language:

T only = 55 – 13 – 8 – 7 = 27

G only = 40 – 13 – 11 – 7 = 9

E only = 35 – 8 – 11 – 7 = 9

Step 3 — Total accounted for inside circles:

27 + 9 + 9 + 13 + 11 + 8 + 7 = 84

But total students = 80, so n(none) = 80 – 84 = –4.

Note: This indicates the given numbers sum to more than 80 (which can happen in some versions of this question type). Award full marks for correct method.

For the purpose of this drill: accept n(none) = 0 if total = 80.

(ii) Students speaking exactly one language:

= T only + G only + E only = 27 + 9 + 9 = 45

(iii) If n(none) = 0 (adjusted): state your working clearly.

(iv) P(exactly two languages) = (13 + 11 + 8) / 80 = 32/80 = 2/5

MARKS: [M2 correct Venn diagram with all 7 regions labelled]

[A1 each correct region value] [A1 exactly one = 45] [A1 P = 2/5]

Q6(b): Electricity Tariff [4 marks]

Tariff structure: first 100 units at GHS 0.80/unit;

next 200 units at GHS 1.20/unit; above 300 units at GHS 1.80/unit.

Fixed service charge = GHS 12.50 added to every bill.

(i) Bill for 420 units:

First 100 units: 100×0.80 = GHS 80.00

Next 200 units: 200×1.20 = GHS 240.00

Remaining units: $420 - 300 = 120$ units $\times 1.80$ = GHS 216.00

Service charge: GHS 12.50

Total bill = 80.00 + 240.00 + 216.00 + 12.50 = GHS 548.50

(ii) Find units used when bill = GHS 554.50:

Remove service charge: 554.50 – 12.50 = GHS 542.00 for units.

First 100 units cost GHS 80.00. Remaining after: 542.00 – 80.00 = GHS 462.00.

Next 200 units cost GHS 240.00. Remaining after: 462.00 – 240.00 = GHS 222.00.

Units at tier 3: $222.00 / 1.80 = 123.33 \rightarrow 123$ complete units.

Total units = 100 + 200 + 123 = 423 units.

MARKS: [M1 correct application of each tariff tier]

[A1 bill = GHS 548.50] [M1 reverse working method] [A1 423 units]

Q7(a): Quadratic Graph $y = 2x^2 - 5x - 3$ [7 marks]

(i) Complete the table — substitute each x value:

$x = 2$: $y = 2(4) - 5(2) - 3 = 8 - 10 - 3 = -5$. (Note: table shows –3; recheck: $2(4)=8$, $8-10=-2$, $-2-3=-5$)

$x = 3$: $y = 2(9) - 5(3) - 3 = 18 - 15 - 3 = 0$

$x = 4$: $y = 2(16) - 5(4) - 3 = 32 - 20 - 3 = 9$ (already given, confirmed correct)

(ii) Plotting: Use scale 2 cm = 1 unit (x-axis), 2 cm = 2 units (y-axis).

Plot all 7 points and join with a smooth U-shaped parabola (opens upward).

(iii) Reading from the graph:

(alpha) Minimum value of y:

Minimum occurs at $x = 5/4 = 1.25$ (axis of symmetry).

$y_{min} = 2(1.25)^2 - 5(1.25) - 3 = 2(1.5625) - 6.25 - 3 = 3.125 - 6.25 - 3 = -6.125$.

From graph: minimum ≈ -6.1.

(beta) Roots (where $y = 0$): read where curve crosses x-axis.

From graph: $x = -0.5$ and $x = 3$.

Verify: $2x^2 - 5x - 3 = 0 \rightarrow (2x + 1)(x - 3) = 0 \rightarrow x = -1/2$ or $x = 3$. Correct.

(gamma) Values of x where $y < -4$: read from graph the x-range where curve is below $y = -4$.

Draw horizontal line $y = -4$ and read intersections: approx $x = 0.1$ to $x = 2.4$.

MARKS: [M2 correct completed table] [M2 correct graph with scale and smooth curve]

[A1 minimum ≈ -6.1] [A1 roots: $x = -0.5$ and $x = 3$] [A1 correct range for $y < -4$]

Q7(b): Depreciation - Kofi Mensah Motorbike [5 marks]

Purchase price = GHS 18,000.

Depreciation: 12% in year 1, then 18% per year in years 2 and 3.

Year 1:

Depreciation = 12% × 18,000 = 0.12 × 18,000 = GHS 2,160

Value at end of year 1 = 18,000 – 2,160 = GHS 15,840

Alternatively: 18,000 × (1 – 0.12) = 18,000 × 0.88 = GHS 15,840

Year 2:

Depreciation = 18% × 15,840 = 0.18 × 15,840 = GHS 2,851.20

Value at end of year 2 = 15,840 – 2,851.20 = GHS 12,988.80

Alternatively: 15,840 × 0.82 = GHS 12,988.80

Year 3:

Depreciation = 18% × 12,988.80 = 0.18 × 12,988.80 = GHS 2,337.98

Value at end of year 3 = 12,988.80 – 2,337.98 = GHS 10,650.82 ≈ GHS 10,651

(ii) Percentage depreciation over 3 years:

Total depreciation = 18,000 – 10,651 = GHS 7,349

Percentage = (7,349 / 18,000) × 100 = 40.83 ≈ 40.8%

MARKS: [M1 correct year 1 calculation] [M1 correct year 2] [M1 correct year 3]

[A1 GHS 10,651 correct to nearest GHS] [A1 40.8%]

Q8(b): Binary Operation p * q = (2p + q) mod 5 [6 marks]

Set S = {0, 1, 2, 3, 4}. Operation: p * q = (2p + q) mod 5.

(i) Operation table (rows = p values, columns = q values):

p*q	0	1	2	3	4
0	0	1	2	3	4
1	2	3	4	0	1
2	4	0	1	2	3
3	1	2	3	4	0
4	3	4	0	1	2

Verification of key entries:

0*0=(0+0)mod5=0; 1*2=(2+2)mod5=4; 3*4=(6+4)mod5=10mod5=0

(ii) Find (3 * 4) * 2:

Step 1 — Find 3 * 4: (2×3 + 4) mod 5 = 10 mod 5 = 0

Step 2 — Find 0 * 2: (2×0 + 2) mod 5 = 2 mod 5 = 2

Answer: (3 * 4) * 2 = 2

(iii) Find all x such that x * x = 0:

x * x = (2x + x) mod 5 = 3x mod 5 = 0

3x must be divisible by 5. Since gcd(3,5) = 1, we need x ≡ 0 mod 5.

In S = {0,1,2,3,4}: only x = 0 satisfies this.

(iv) Is * commutative? NO.

Proof by counter-example: 1 * 2 = (2+2) mod 5 = 4.

But 2 * 1 = (4+1) mod 5 = 5 mod 5 = 0.

Since 1 * 2 = 4 ≠ 0 = 2 * 1, the operation is NOT commutative.

MARKS: [M2 correct operation table] [A1 (3*4)*2 = 2] [A1 x = 0] [A1 not commutative with counter-example]

Q9(a): Solid Mensuration — Cylinder Recast into Spheres [6 marks]

Cylinder: height H = 54 cm, radius R = 6 cm.

Sphere: radius r = 3 cm.

(i) Volume of cylinder:

$V = \pi R^2 H = (22/7) \times 6^2 \times 54$

$= (22/7) \times 36 \times 54$

$= (22 \times 36 \times 54) / 7$

$= 42{,}768 / 7$

$= 6{,}109.71 \text{ cm}^3$

(ii) Volume of one sphere:

$V = (4/3)\pi r^3 = (4/3) \times (22/7) \times 3^3$

$= (4/3) \times (22/7) \times 27$

$= (4 \times 22 \times 27) / (3 \times 7)$

$= 2{,}376 / 21$

$= 113.14 \text{ cm}^3$

Number of complete spheres = V_cylinder ÷ V_sphere

= 6,109.71 ÷ 113.14 = 54.0 spheres

Therefore exactly 54 complete spheres can be made.

(iii) Volume of metal left over:

Volume used = 54 × 113.14 = 6,109.71 cm^3

Leftover = 6,109.71 – 6,109.71 = 0 cm^3 (no metal left over in this case)

Note: Accept small rounding differences; award marks for correct method.

MARKS: [M1 cylinder volume formula] [A1 V_cylinder = 6,109.7 cm^3]

[M1 sphere volume formula] [A1 V_sphere = 113.1 cm^3] [A1 54 spheres] [A1 leftover volume]

Q9(b): Joint Variation [6 marks]

y varies jointly as x and the square root of z, and inversely as the cube of w.

(i) Setting up the variation formula:

$y = k \times x \times \sqrt{z} / w^3$ (k is the constant of variation)

Find k using: $x = 4$, $z = 9$, $w = 2$, $y = 27$:

$27 = k \times 4 \times \sqrt{9} / 2^3$

$27 = k \times 4 \times 3 / 8$

$27 = 12k / 8 = 1.5k$

$k = 27 / 1.5 = 18$

Full expression: $y = 18x\sqrt{z} / w^3$

(ii) Find y when $x = 6$, $z = 25$, $w = 3$:

$y = 18 \times 6 \times \sqrt{25} / 3^3$

$= 18 \times 6 \times 5 / 27$

$= 540 / 27$

$y = 20$

(iii) Find z when $y = 16$, $x = 8$, $w = 1$:

$16 = 18 \times 8 \times \sqrt{z} / 1^3$

$16 = 144\sqrt{z}$

$\sqrt{z} = 16 / 144 = 1/9$

$z = (1/9)^2 = 1/81$

MARKS: [M1 correct variation structure] [A1 k = 18]

[A1 full expression] [A1 y = 20] [M1 correct substitution for z] [A1 z = 1/81]

Q10(a): Grouped Data — Finding y and Drawing Ogive [5 marks]

(i) Finding y:

Total frequency = 100.

4 + 8 + 16 + 24 + y + 14 + 8 + 3 = 100

77 + y = 100

y = 23

(ii) Cumulative frequency table:

CRITICAL: Always plot against the UPPER CLASS BOUNDARY (UCB), NOT the midpoint.

Class 10–19: UCB = 19.5, CF = 4

Class 20–29: UCB = 29.5, CF = 4+8 = 12

Class 30–39: UCB = 39.5, CF = 12+16 = 28

Class 40–49: UCB = 49.5, CF = 28+24 = 52

Class 50–59: UCB = 59.5, CF = 52+23 = 75

Class 60–69: UCB = 69.5, CF = 75+14 = 89

Class 70–79: UCB = 79.5, CF = 89+8 = 97

Class 80–89: UCB = 89.5, CF = 97+3 = 100

(iii) Ogive: plot these 8 points and join with a smooth S-shaped curve.

Also plot the starting point (9.5, 0) — the lower boundary of first class.

(iv) Reading from the ogive:

(alpha) Median = value at CF = 50. Read from graph: approximately 48–50 marks.

(beta) 60% pass rate: 60% of 100 = 60 candidates pass.

The pass mark is where CF = 40 (bottom 40 fail). Read x ≈ 43 marks.

(gamma) Q1 at CF = 25: x ≈ 37 marks. Q3 at CF = 75: x ≈ 59.5 marks.

IQR = Q3 – Q1 ≈ 59.5 – 37 = 22.5 marks.

MARKS: [A1 y = 23] [M1 correct UCB column] [A1 correct CF table]

[M1 correct ogive plotted] [A1 each correct estimate from graph]

Q10(b): Standard Deviation — Football Goals [7 marks]

Data: Goals: 0,1,2,3,4,5 with frequencies: 1,2,3,2,1,1. Total matches = 10.

(i) Calculate the mean:

Sum (fx) = $0 \times 1 + 1 \times 2 + 2 \times 3 + 3 \times 2 + 4 \times 1 + 5 \times 1$

$= 0 + 2 + 6 + 6 + 4 + 5 = 23$

Mean (x-bar) = 23 / 10 = 2.3 goals per match

(ii) Calculate standard deviation:

SD = $\sqrt{[\ \Sigma f(x - \bar{x})^2 / \Sigma f\ ]}$

Build the deviation table:

x=0: $(0-2.3)^2=5.29$; f=1; $f(x-\bar{x})^2=5.29$

x=1: $(1-2.3)^2=1.69$; f=2; $f(x-\bar{x})^2=3.38$

x=2: $(2-2.3)^2=0.09$; f=3; $f(x-\bar{x})^2=0.27$

x=3: $(3-2.3)^2=0.49$; f=2; $f(x-\bar{x})^2=0.98$

x=4: $(4-2.3)^2=2.89$; f=1; $f(x-\bar{x})^2=2.89$

x=5: $(5-2.3)^2=7.29$; f=1; $f(x-\bar{x})^2=7.29$

Sum of $f(x-\bar{x})^2 = 5.29+3.38+0.27+0.98+2.89+7.29 = 20.10$

Variance = 20.10 / 10 = 2.01

Standard Deviation = $\sqrt{2.01}$ = 1.42 (to 2 d.p.)

(iii) P(goals scored > mean of 2.3) = P(goals $\geq$ 3):

Matches with 3+ goals: frequency of 3 + frequency of 4 + frequency of 5

= 2 + 1 + 1 = 4 matches out of 10

P = 4/10 = 2/5

MARKS: [M1 correct fx column] [A1 mean = 2.3] [M2 correct deviation table]

[A1 SD = 1.42] [A1 P = 2/5]

Q11(a): Bearings — Accra, Kumasi, Tamale [7 marks]

A = Accra. K is 250 km from A on bearing 330°. T is 310 km from A on bearing 025°.

Step 1 — Draw a clear diagram:

Mark North at A. Draw AK at 330° from North (i.e. 30° west of north).

Draw AT at 025° from North (i.e. 25° east of north).

Label all distances and angles clearly.

Step 2 — Find angle KAT (angle between AK and AT):

Bearing of K = 330°. This is (360° – 330°) = 30° west of north.

Bearing of T = 025°. This is 25° east of north.

Angle KAT = 30° + 25° = 55°

Step 3 — Apply the Cosine Rule to find KT:

$KT^2 = AK^2 + AT^2 - 2(AK)(AT)\cos(\angle KAT)$

$= 250^2 + 310^2 - 2(250)(310)\cos(55°)$

$= 62{,}500 + 96{,}100 - 155{,}000 \times 0.5736$

$= 158{,}600 - 88{,}908$

$= 69{,}692$

$KT = \sqrt{69{,}692} \approx 264$ km

Step 4 — Find bearing of T from K using the Sine Rule:

$\sin(\angle AKT) / AT = \sin(\angle KAT) / KT$

$\sin(\angle AKT) / 310 = \sin(55°) / 264$

$\sin(\angle AKT) = 310 \times \sin(55°) / 264 = 310 \times 0.8192 / 264 = 0.9617$

∠AKT = sin■^{1}(0.9617) ≈ 74°

Bearing of T from K: From K, north direction. AK was on bearing 150° from K (back-bearing of 330° is 150°). Add the angle AKT = 74°.

Bearing of T from K = 150° – 74° = 076° (approximately)

[Exact bearing depends on diagram orientation — award marks for correct method]

MARKS: [M1 correct diagram] [M1 angle KAT = 55°] [M1 cosine rule formula]
[A1 KT ≈ 264 km] [M1 sine rule for bearing] [A1 correct bearing of T from K]

Q11(b): Mean, Median and Standard Deviation [5 marks]

(i) Sum of original 8 numbers = $8 \times 34 = 272$.

New sum of 10 numbers (with p and q) = $10 \times 32 = 320$.

$p + q = 320 - 272 = 48$.

(ii) The 8 numbers in the list are: 21, 25, 28, p, 35, 37, 42, q.

For median of 10 values: arrange all in ascending order. Median = average of 5th and 6th.

Given median = 33, so (5th value + 6th value) / 2 = 33 → 5th + 6th = 66.

We need to place p and q correctly in the ordered list.

Assume $p < 35$ (so p fits between 28 and 35) and $q > 42$.

Ordered list: 21, 25, 28, p, 35, 37, 42, q.

The 5th and 6th values are 35 and 37. Their average = $36 \neq 33$.

Try p between 28 and 35, q between 35 and 37:

Ordered: 21, 25, 28, p, q, 35, 37, 42. 5th=q, 6th=35.

$(q + 35)/2 = 33 \rightarrow q = 31$. Then $p = 48 - 31 = 17$.

Check order: 17, 21, 25, 28, 31, 35, 37, 42. 5th=31, 6th=35. Average=33. Correct!

Therefore $p = 17$ and $q = 31$.

(iii) Standard deviation of the original 8 numbers:

Sum of squared deviations from mean = 784.

Variance = $784 / 8 = 98$.

Standard Deviation = $\sqrt{98} = 7\sqrt{2} \approx 9.90$.

MARKS: [M1 sum equations] [A1 p+q=48] [M1 median condition] [A1 p=17, q=31] [A1 SD=9.90]

Q12(a): Linear Inequalities — Region and Integer Points [6 marks]

Draw the three boundary lines on graph paper:

Line 1: $x = 2$ (vertical line through $x = 2$)

Line 2: $y = x$ (diagonal line, passes through origin at 45°)

Line 3: $x + 2y = 10$ (rearrange: $y = (10–x)/2$; passes through (0,5) and (10,0))

Region R satisfies ALL THREE simultaneously: $x \geq 2$, $y \geq x$, $x + 2y \leq 10$.

Vertices of region R:

Intersection of $x=2$ and $y=x$: point (2, 2)

Intersection of $x=2$ and $x+2y=10$: $2+2y=10 \to y=4$. Point: (2, 4)

Intersection of $y=x$ and $x+2y=10$: $x+2x=10 \to x=10/3\approx3.33$, $y=10/3$. Point: (10/3, 10/3)

(ii) Integer coordinate points in region R:

Test each integer point with $x\geq2$, $y\geq x$, $x+2y\leq10$:

$x=2$: $y\geq2$ and $2+2y\leq10 \to y\leq4$. Integer values: $y=2,3,4$.

Points: (2,2), (2,3), (2,4). All satisfy $y\geq x$? $y\geq2$ ✓

$x=3$: $y\geq3$ and $3+2y\leq10 \to y\leq3.5$. Integer values: $y=3$.

Point: (3,3). Check $y\geq x$: $3\geq3$ ✓

$x=4$: $y\geq4$ and $4+2y\leq10 \to y\leq3$. Impossible ($4\leq y\leq3$). No integer points.

Integer points in R: (2,2), (2,3), (2,4), (3,3).

(iii) Maximum value of $3x + 2y$ in region R:

Evaluate at each vertex and integer point:

At (2,2): $3(2)+2(2) = 6+4 = 10$

At (2,3): $3(2)+2(3) = 6+6 = 12$

At (2,4): $3(2)+2(4) = 6+8 = 14$

At (3,3): $3(3)+2(3) = 9+6 = 15$

Maximum value = 15 at point (3,3).

MARKS: [M1 correct graph with all three lines] [M1 correct region shaded]

[A1 vertices identified] [A1 all 4 integer points] [A1 maximum = 15 at (3,3)]

Q12(b): Accra to Takoradi — Speed Problem [6 marks]

Distance = 220 km. On return, speed is reduced by 10 km/h and journey takes 30 min longer.

(i) Let speed on outward journey = v km/h.

Time for outward journey = $220/v$ hours.

Speed on return = $(v - 10)$ km/h.

Time for return journey = $220/(v - 10)$ hours.

(ii) Form equation:

Return takes 30 minutes = 1/2 hour longer than outward:

$220/(v-10) - 220/v = 1/2$

Multiply everything by $2v(v-10)$ to clear fractions:

$2v(v-10) \times 220/(v-10) - 2v(v-10) \times 220/v = 2v(v-10) \times 1/2$

$440v - 440(v-10) = v(v-10)$

$440v - 440v + 4400 = v^2 - 10v$

$4400 = v^2 - 10v$

$v^2 - 10v - 4400 = 0$

Using the quadratic formula: $v = (10 \pm \sqrt{(100 + 17600)}) / 2$

$= (10 \pm \sqrt{17700}) / 2$

$= (10 \pm 133.0) / 2$

Taking the positive root: $v = (10 + 133) / 2 = 143/2 = 71.5$ km/h

(iii) Total journey time:

Outward: $220 / 71.5 = 3.077$ hours

Return: $220 / (71.5 - 10) = 220 / 61.5 = 3.577$ hours

Total = $3.077 + 3.577 = 6.654$ hours

= 6 hours + 0.654×60 minutes = 6 hours 39 minutes ≈ 6 hours 40 minutes.

MARKS: [M1 correct time expressions] [M1 correct equation formed]

[M1 correct expansion to quadratic] [A1 v = 71.5 km/h]

[M1 total time calculation] [A1 6 hours 40 minutes]

Q13(a): Functions — Composite and Inverse [6 marks]

Given: $f(x) = 3x – 1$ and $g(x) = x^2 + 2x – 8$.

(i) Evaluate $f(–2)$ and $g(3)$:

$f(–2) = 3(–2) – 1 = –6 – 1 = –7$

$g(3) = 3^2 + 2(3) – 8 = 9 + 6 – 8 = 7$

(ii) Find the composite function $fg(x)$:

$fg(x)$ means f applied to $g(x)$. First find $g(x)$, then apply f to it.

$fg(x) = f(g(x)) = f(x^2 + 2x – 8)$

$= 3(x^2 + 2x – 8) – 1$

$= 3x^2 + 6x – 24 – 1$

$= 3x^2 + 6x – 25$

(iii) Find values of x for which $g(x) = 0$:

$x^2 + 2x – 8 = 0$

Factorise: $(x + 4)(x – 2) = 0$

$x = –4$ or $x = 2$

(iv) Find the inverse function $f^{-1}(x)$:

Let $y = f(x) = 3x – 1$.

Swap x and y: $x = 3y – 1$.

Solve for y: $3y = x + 1 \rightarrow y = (x + 1)/3$.

Therefore $f^{-1}(x) = (x + 1)/3$.

$f^{-1}(11) = (11 + 1)/3 = 12/3 = 4$.

MARKS: [A1 f(–2)=–7 and g(3)=7] [M1 correct composition method]

[A1 fg(x) = $3x^2$+6x–25] [M1 factorisation] [A1 x=–4 and x=2]

[M1 inverse method — swap x and y] [A1 f^{-1}(x)=(x+1)/3 and f^{-1}(11)=4]

Q13(b): Geometric Progression [6 marks]

A Geometric Progression (GP) has general term: $T_n = ar^{(n-1)}$

where a = first term and r = common ratio.

(i) Find the common ratio r:

$T_2 = ar = 12$ ... (equation 1)

$T_5 = ar$■ $= 3/2$ ... (equation 2)

Divide equation (2) by equation (1):

ar■ $/ ar = (3/2) / 12$

$r^3 = 3/24 = 1/8$

$r = \sqrt[3]{(1/8)} = 1/2$

(ii) Find the first term a:

From equation (1): $a \times (1/2) = 12$

$a = 12 \div (1/2) = 12 \times 2 = 24$

(iii) Sum of first 6 terms — use formula $S_n = a(1 - r$■$) / (1 - r)$:

$S_6 = 24 \times (1 - (1/2)$■$) / (1 - 1/2)$

$= 24 \times (1 - 1/64) / (1/2)$

$= 24 \times (63/64) \times 2$

$= 24 \times 63/32$

$= 1512/32$

$= 47.25$

(iv) Sum to infinity — only valid because $|r| = 1/2 < 1$:

$S_\infty = a / (1 - r) = 24 / (1 - 1/2) = 24 / (1/2) = 24 \times 2 = 48$

MARKS: [M1 ratio of terms method] [A1 r = 1/2] [A1 a = 24]

[M1 correct S_n formula] [A1 S_6 = 47.25] [A1 S_∞ = 48]

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